

# A Review of UAV Path Planning Algorithms

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## Abstract

With the rapid development of unmanned aerial vehicle (UAV) technology, its applications have widely penetrated into various fields such as military, civil and commercial sectors. As a key link for UAVs to perform tasks efficiently and safely, path planning has attracted extensive attention. This paper conducts a systematic review of UAV path planning algorithms. Firstly, it expounds the research background and significance, and points out that complex environments and diversified tasks put forward high requirements for path planning. Then, it introduces in detail the principles and characteristics of path planning algorithms. Subsequently, it analyzes the current challenges faced by UAV path planning, such as adaptation to complex environments, multi-UAV collaboration, and real-time re-planning in dynamic environments. Finally, it looks forward to the future development trends such as algorithm fusion, application of deep learning, and multi-objective optimization. This review aims to provide a reference for the further research and application of UAV path planning algorithms.

## Keywords

UAV; path planning; traditional algorithms; intelligent optimization algorithms.

## 1. Introduction

With the increasing complexity of application scenarios, the flight environments faced by unmanned aerial vehicles (UAVs) have become increasingly harsh. In urban environments, there are numerous high-rise buildings and severe electromagnetic interference; in mountainous areas, the terrain is rugged and meteorological conditions are variable; in marine environments, the influence of wind fields is significant. All these pose great challenges to UAV path planning. UAV path planning not only needs to meet basic constraints such as obstacle avoidance, endurance, and time, but also needs to achieve multi-objective optimization according to different task requirements. For example, in logistics distribution, the shortest path is pursued to reduce costs; in emergency rescue, the fastest arrival speed is pursued to gain rescue time; in agricultural plant protection, the maximum coverage area is pursued to improve operation quality.

At present, although various path planning algorithms have been proposed and applied, there are still deficiencies in terms of real-time performance in complex dynamic environments, coordination of multi-UAV collaborative operations, and optimization accuracy under multi-constraint conditions. For instance, in emergency disaster rescue, UAVs need to respond quickly and plan paths that avoid secondary disaster areas, but existing algorithms may fail to meet real-time requirements due to high computational complexity. In multi-UAV collaborative distribution, how to reasonably allocate paths to avoid collisions and improve overall efficiency remains an urgent problem to be solved.

Therefore, in-depth research on UAV path planning algorithms and continuous optimization of their performance to enable them to adapt to various complex environments and task requirements are of vital significance for promoting the further development and wide application of UAV technology. This can not only improve the autonomous operation capability

and task execution efficiency of UAVs, reduce labor costs and operation risks, but also provide strong support for technological innovation and industrial upgrading in related fields.

## 2. Research Status

In recent years, a large number of path planning algorithms have emerged at home and abroad. Various algorithms have their own advantages and disadvantages, and are applicable to different scenarios. In 2011, Y. Volkan Pehlivanoglu [1] proposed a multi-frequency vibrational genetic algorithm (mVGA), which emphasizes a new mutation application strategy. Clustering methods and the concept of Voronoi diagrams are used in the initial population stage of the mVGA process, which can be applied to solve the path planning problem of autonomous unmanned aerial vehicles (UAVs). In 2014, Min Yao and Min Zhao [2] put forward a model predictive control (MPC) algorithm to determine the optimal or suboptimal path with the minimum total cost. The MPC algorithm is a rolling optimization feedback algorithm. It is used online instead of one-time offline to plan the UAV path in several steps, so as to dynamically avoid sudden and moving threats. In 2017, Manh Duong Phung [3] proposed an improved discrete particle swarm optimization (DPSO) algorithm to solve the TSP problem, which improves the algorithm performance by using deterministic initialization, random mutation and edge exchange. In 2019, Na Lin [4] proposed an improved artificial potential field method with adaptive weights and chaos strategy, which accelerates the convergence process of bat position update, avoids falling into local optimum, significantly improves the success rate of searching for a suitable track, and shortens the convergence time. Gatij Jain [5] proposed a new method to solve the 3D track planning problem of UAVs while maintaining the coordination of target selection. A new algorithm based on the multi-verse optimizer (MVO) is applied to this problem to find the optimal path cost with the minimum average execution time. Zhang Xiangyin [6] proposed an enhanced fruit fly optimization algorithm (QFOA) based on quantum theory to accelerate the convergence of the algorithm and avoid falling into local optimum. When a quantum Delta potential well is established near the position of the fruit fly population, QFOA introduces a quantum behavior-based search mechanism into the original olfactory-based search process of FOA. In the process of fruit flies searching for and moving towards food sources, they follow the wave function properties of the Delta potential well rather than Newtonian mechanics. By using the probability and uncertainty of quantum theory, the proposed QFOA can effectively overcome the shortcomings of premature convergence and easy falling into local optimum. Kun Li [7] proposed an improved fruit fly optimization algorithm (ORPFOA) to solve the path planning problem in the initial task sequence and the new task sequence after task changes. The optimal reference point and distance cost matrix are used to realize the fast solution and high-precision optimization of the optimal flight path. Chengzhi Qu [8] proposed a new hybrid algorithm HSGWO-MSOS by combining the simplified grey wolf optimizer (SGWO) and the modified symbiotic organisms search algorithm (MSOS), which effectively combines the exploration and exploitation capabilities. The stages of the GWO algorithm are simplified to accelerate the convergence speed and retain the exploration ability of the population. The nominal stage of the SOS algorithm is improved and integrated with GWO to enhance the exploitation ability of the algorithm. Zhou Yaoming [9] proposed a biologically inspired 3D path planning algorithm to solve the problem of UAV dynamic obstacle avoidance path planning in the case of unknown environment maps. Compared with other path planning algorithms, this algorithm has the advantages of fast path planning speed and fewer path points, and can achieve the effect of low-latency real-time path planning. Mesquita Ricardo [10] presented a UAV track planning optimization algorithm based on particle swarm optimization. This path planning optimization algorithm aims to manage the distance and flight time of UAVs, and applies optimization and randomness technologies to overcome the shortcomings of traditional systems. Chen Yueyi [11] aimed at the shortcomings of the original wolf pack

algorithm, such as poor local search ability, fixed step size and slow convergence speed. Based on the luciferin guidance mechanism in the artificial firefly swarm optimization algorithm, an improved wolf pack algorithm is proposed, which improves the search mode in the wandering behavior, and uses the teaching-learning optimization algorithm with feedback mechanism to update the individual positions in the wolf pack. Wan Yuting [12] introduced an accurate swarm intelligence search method based on improved ant colony optimization to obtain the optimal 3D flight path of UAVs, which improves the global and local search capabilities through priority search direction and random neighborhood search mechanism. Du Yuwen [13] addressed the problems of long calculation time and large memory occupation of UAV obstacle avoidance path planning algorithms in collaborative tasks, and proposed a method combining the A\* algorithm with task allocation algorithms to realize a faster and more effective path planning method. Lixia Deng [14] focused on the problem that the traditional particle swarm optimization algorithm is fast and efficient but easy to fall into local optimum. It is proposed to combine the PSO algorithm with the genetic algorithm (GA), set dynamic inertia weight, add the sigmoid function to improve the crossover and mutation probabilities of the genetic algorithm, and change the selection mode to apply it to UAV 3D path planning.

### 3. Classification and Characteristics of UAV Path Planning Algorithms

#### 3.1. Traditional Path Planning Algorithms

##### 3.1.1. Dijkstra Algorithm

The Dijkstra algorithm, proposed by Dutch computer scientist Edsger W. Dijkstra in 1956, is a classic shortest path search algorithm. In the field of UAV path planning, it is often used to solve the problem of finding the shortest path from the starting point to the destination in static environments with no obstacles or with known obstacles.

The core idea of this algorithm is based on a greedy strategy: starting from the initial point, it gradually explores all reachable surrounding nodes. At each step, it selects the node that is closest to the starting point and has not been visited yet, takes this node as an intermediate point, and updates the distance from its adjacent nodes to the starting point. This process continues until the destination is found or all reachable nodes are traversed.

The advantage of the Dijkstra algorithm is that it can guarantee finding the global optimal solution, with a stable and reliable calculation process. However, it has obvious shortcomings: its time complexity is relatively high, leading to low computational efficiency in large-scale environments or complex terrains, which makes it difficult to meet real-time requirements.

##### 3.1.2. A\* Algorithm

The A\* algorithm is a heuristic search algorithm improved from the Dijkstra algorithm. It introduces a heuristic function, which estimates the distance from the current node to the destination and guides the search process to move in directions more likely to approach the destination, thereby improving search efficiency. Compared with the Dijkstra algorithm, the A\* algorithm generally has a faster search speed and can find a relatively optimal path in a shorter time. However, the performance of this algorithm largely depends on the design of the heuristic function; an inappropriate choice of the heuristic function may lead to reduced search efficiency or even failure to find the optimal path.

The A\* algorithm is a grid-based intelligent heuristic search algorithm. It represents the search space in the form of a grid, with the center points or vertices of the grid serving as waypoints. By searching for the waypoint with the smallest cost function value in the neighborhood, it gradually searches from the starting point to the target point. Finally, it generates the optimal path by backtracking the parent node of the current node in reverse. Among them, the

waypoints that have been calculated but not yet expanded are stored in the "OPEN" list, and the expanded nodes are stored in the "CLOSE" list. The expression of the cost function is:

$$f(x) = g(x) + h(x) \quad (1)$$

In the formula,  $g(x)$  represents the actual cost from the starting point to the current node;  $h(x)$  is the heuristic function, which denotes the estimated cost from the current node to the target point.

### 3.1.3. Artificial Potential Field Method

The artificial potential field method was proposed by Khatib in 1986, initially for robot obstacle avoidance, and later extended to UAV path planning. Its core principle is to simulate the motion of an object under force in a physical field: the target point generates an attractive force on the UAV, while obstacles generate a repulsive force, and the UAV moves toward the target point under the action of the resultant force.

Attractive force: It is positively correlated with the distance from the UAV to the target point.

$$F_{att} = k \cdot \rho \quad (2)$$

In the formula,  $k$  is a coefficient, and  $\rho$  is the distance.

Repulsive force: It is negatively correlated with the distance from the UAV to the obstacle.

$$F_{rep} = \eta \cdot \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right)^2 \quad (3)$$

In the formula,  $\eta$  is a coefficient, and  $\rho_0$  is the influence range of the obstacle.

The artificial potential field method has advantages such as small computational load, strong real-time performance, and the ability to handle dynamic obstacles (by updating the potential field in real time). However, it also has drawbacks, including the problem of local minima (where the resultant force is zero, causing the UAV to stagnate) and potential interference from the repulsive force of obstacles near the target point.

## 3.2. Intelligent Optimization Algorithms

### 3.2.1. Genetic Algorithm

The genetic algorithm (GA) was proposed in 1975 by American scientist Professor John Holland, based on the theory of "natural selection and survival of the fittest" from Darwin's theory of evolution. It searches for the optimal solution in the solution space by simulating processes such as gene selection, crossover, and mutation.

In genetic algorithms, each potential solution to a problem is regarded as a "chromosome" (usually represented in forms such as binary strings or real-number arrays), and multiple chromosomes form a "population". The algorithm simulates the evolutionary process through the following steps, see Table 1.

Table 1 Three Core Operations of Genetic Algorithm

Core Operation	Description of Operation	Function
Selection	Based on the fitness value of chromosomes (reflecting the quality of	Retain high-quality individuals and pass their

	<p>solutions), individuals are selected from the population as parents with a certain probability. Individuals with higher fitness have a greater probability of being selected, such as roulette wheel selection, tournament selection, etc.</p>	<p>genes to the next generation, embodying the evolutionary idea of "survival of the fittest".</p>
Crossover	<p>Select two parent chromosomes and exchange partial gene segments at random or specific positions to generate new offspring chromosomes. Common methods include single-point crossover, multi-point crossover, uniform crossover (for binary encoding), or arithmetic crossover, BLX crossover (for real-number encoding), etc.</p>	<p>Combine the excellent genes of parents to produce new individuals with potentially better solutions and increase the diversity of the population.</p>
Mutation	<p>Randomly alter one or more genes of a chromosome, such as changing 0 to 1 or 1 to 0 in binary encoding, or randomly perturbing gene values within a certain range in real-number encoding. The mutation probability is usually low.</p>	<p>Introduce new genetic information, prevent the population from falling into local optimum, maintain population diversity, and provide new possibilities for evolution.</p>

Through multiple generations of iteration, the population gradually evolves toward higher fitness, and finally converges to an approximate optimal solution. Its advantages include adaptability, global optimization, and implicit parallelism, with extremely strong global search capability. However, it also has disadvantages such as slow convergence speed and being prone to premature convergence to local optima. Therefore, many improved methods for the basic genetic algorithm have been proposed.

### 3.2.2. Particle Swarm Optimization Algorithm

Particle Swarm Optimization (PSO) is a stochastic optimization technique based on swarm intelligence, proposed by Kennedy and Eberhart in 1995, inspired by the foraging behavior of bird flocks. By simulating information sharing and collaboration among individuals in a group, the algorithm searches for the optimal solution in the solution space. Due to its characteristics such as simple structure and fast convergence speed, it is widely applied in the field of UAV path planning, especially suitable for multi-constraint optimization problems in continuous spaces. In the PSO algorithm, each potential solution is regarded as a "particle", and multiple particles form a "particle swarm". Each particle moves in the solution space, and its state is described by "position" (corresponding to the specific value of the solution) and "velocity" (determining the direction and magnitude of position update). The core of the algorithm is that particles dynamically adjust their movement trajectories by learning from their own historical optimal experience and the global optimal experience of the swarm, gradually approaching the optimal solution.

Velocity update formula:

$$v_i^t = \omega \cdot v_i^{t-1} + c_1 \cdot r_1 \cdot (pbest_i - x_i^{t-1}) + c_2 \cdot r_2 \cdot (gbest_i - x_i^{t-1}) \quad (4)$$

Where:  $v_i^t$  is the velocity of particle  $i$  in the  $t$ -th generation;  $\omega$  is the inertia weight (balancing global and local search);  $c_1$  and  $c_2$  are learning factors (usually set to 2, controlling the intensity

of learning from individual/group experience);  $r_1$  and  $r_2$  are random numbers in the interval  $[0,1]$  (increasing search randomness);  $pbest_i$  is the individual historical optimal position of particle  $i$ ;  $gbest_i$  is the global optimal position of the entire swarm;  $x_i^{t-1}$  is the position of particle  $i$  in the  $(t-1)$ -th generation.

Position update formula:

$$x_i^t = x_i^{t-1} + v_i^t \quad (5)$$

The Particle Swarm Optimization algorithm is derived from the simulation of the foraging behavior of bird flocks. Each particle moves in the solution space and adjusts its movement direction and speed by learning from its own and the swarm's experience to find the optimal solution.

The advantages of this algorithm include a simple structure, ease of implementation, and relatively fast convergence. However, when dealing with high-dimensional problems with complex constraints, it is prone to premature convergence, resulting in low search accuracy.

### 3.2.3. Ant Colony Algorithm

The ant colony algorithm is a swarm intelligence optimization algorithm that simulates the foraging behavior of ants in nature, proposed by Italian scholar Dorigo in 1991. By simulating the mechanism of ants releasing and perceiving pheromones on paths, this algorithm realizes the search for the optimal path, and is particularly suitable for solving combinatorial optimization problems.

In nature, when ants forage, they leave pheromones on the path, and other ants tend to choose paths with higher pheromone concentrations; at the same time, pheromones will volatilize over time. Through the positive feedback mechanism of "pheromone accumulation and volatilization", the ant colony will eventually converge to the shortest path from the ant nest to the food source.

The probability that ant  $k$  selects the next node  $j$  from node  $i$  is:

$$p_{ij}^k(t) = \begin{cases} \frac{[\tau_{ij}(t)]^\alpha \cdot (\eta_{ij})^\beta}{\sum_{s \in allowed_k} [\tau_{ij}(t)]^\alpha \cdot (\eta_{ij})^\beta}, & \text{if } j \in allowed_k \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

In the formula,  $allowed_k$  represents the set of nodes unvisited by ant  $k$  (to avoid repeated visits);  $\alpha$  is the pheromone heuristic factor (controlling the weight of pheromone influence);  $\beta$  is the expected heuristic factor (controlling the weight of path distance influence);  $\eta_{ij}=1/d_{ij}$  is the heuristic function (where  $d_{ij}$  is the distance between nodes  $i$  and  $j$ ; the shorter the distance, the larger  $\eta_{ij}$ ).

The ant colony algorithm simulates the behavior of ants releasing pheromones during their search for food. Through the mechanism of pheromone accumulation and volatilization, it guides the ant colony to find the shortest path from the ant nest to the food source.

The ant colony algorithm has strong robustness and distributed computing capabilities, making it suitable for solving combinatorial optimization problems. However, it has issues such as slow convergence speed, susceptibility to falling into local optima, and its performance is greatly affected by parameter settings.

### 3.2.4. Simulated Annealing Algorithm

Simulated Annealing (SA) is a stochastic optimization algorithm inspired by the physical annealing process. It was proposed by Metropolis et al. in 1953 and later applied to combinatorial optimization problems by Kirkpatrick et al. in 1983. Its core idea is to simulate the law that substances gradually tend to the lowest energy state as temperature decreases during annealing, performing probabilistic searches in the solution space to find the global optimal solution. Due to its ability to effectively escape local optima and strong robustness, the algorithm is widely used in path planning, function optimization, machine learning, and other fields.

Physical annealing refers to the process of heating a solid to a high temperature and then cooling it slowly: at high temperatures, atoms have high kinetic energy and can move freely (corresponding to random searches in the algorithm); as the temperature decreases, atoms gradually tend to an ordered state with the lowest energy (corresponding to the algorithm converging to the optimal solution).

(1) Energy state: In the algorithm, it corresponds to the "objective function value" (such as path length, cost, etc., which need to be minimized or maximized);

(2) Temperature: A parameter controlling the randomness of the search. At high temperatures, the probability of accepting worse solutions is high (global exploration); at low temperatures, only better solutions are accepted (local refinement).

## 3.3. Intelligent Optimization Algorithms

### 3.3.1. Fuzzy Logic Algorithm

The fuzzy logic algorithm is based on fuzzy set theory and performs path planning by simulating human fuzzy decision-making processes. It does not require an accurate mathematical model and can handle uncertain environmental information.

The advantages of the fuzzy logic algorithm include high flexibility and strong adaptability to environmental changes. However, its control rules rely on expert experience for design, and decision-making inaccuracies may occur in complex environments.

## 4. Challenges in UAV Path Planning

### 4.1. Path Planning in Complex Environments

With the continuous expansion of UAV application scenarios, the environments they face are increasingly complex, such as urban canyons and mountainous jungles. These environments contain numerous static and dynamic obstacles, along with significant terrain undulations, posing huge challenges to path planning. How to quickly and accurately plan safe and feasible paths in complex environments is one of the current research difficulties.

### 4.2. Multi-UAV Collaborative Path Planning

In many practical applications, multiple UAVs are required to collaborate to complete tasks, such as joint reconnaissance and collaborative delivery. Multi-UAV collaborative path planning needs to consider issues like collision avoidance between UAVs, task allocation, and resource sharing, making its complexity much higher than that of single-UAV path planning. How to achieve efficient collaboration among multiple UAVs and improve task execution efficiency is an urgent problem to be solved.

### 4.3. Real-Time Path Replanning in Dynamic Environments

During flight, UAVs may encounter unexpected situations, such as suddenly appearing obstacles or UAV malfunctions, requiring real-time path replanning. Real-time path replanning demands algorithms with fast response capabilities to generate new feasible paths within a

short time. However, many existing algorithms struggle to meet real-time requirements in dynamic environments.

## 5. Future Development Trends

### 5.1. Integrating Advantages of Multiple Algorithms

A single path planning algorithm often has limitations. Future research will focus more on integrating multiple algorithms to fully leverage their advantages. For example, combining intelligent optimization algorithms with traditional path planning algorithms: using the global search capability of intelligent optimization algorithms to find better solutions, then applying traditional algorithms for local optimization to improve path accuracy and efficiency.

### 5.2. Path Planning Based on Deep Learning

Deep learning has achieved great success in fields such as image recognition and speech processing, and its application in UAV path planning has broad prospects. Through deep learning methods, UAVs can learn environmental features and laws from large amounts of environmental data to achieve autonomous path planning. Deep learning-based path planning algorithms have stronger adaptability and generalization ability, enabling better handling of complex and unknown environments.

### 5.3. Path Planning Considering Multi-Objective Optimization

In practical applications, UAV path planning often needs to consider multiple objectives, such as the shortest path, minimum energy consumption, and shortest time. Future research will focus more on multi-objective optimization problems, designing reasonable multi-objective optimization algorithms to balance multiple objectives and find the optimal compromise path.

## 6. Conclusion

UAV path planning algorithms are a core component of UAV technology, and their performance directly affects the effectiveness of UAV task execution. This paper reviews traditional path planning algorithms, intelligent optimization algorithms, and other path planning algorithms, analyzes the characteristics and applicable scenarios of each type of algorithm, and discusses the challenges and future development trends in UAV path planning.

In the future, with the continuous development of UAV technology and the expansion of application scenarios, the requirements for path planning algorithms will become increasingly high. It is necessary to further study the principles and performance of various algorithms, explore new algorithm integration methods and technical means, to improve the efficiency, safety, and adaptability of UAV path planning, and provide strong technical support for the wide application of UAVs.

## References

- [1] Pehlivanoglu V Y: A New Vibrational Genetic Algorithm Enhanced with a Voronoi Diagram for Path Planning of Autonomous UAV, *Aerospace Science and Technology*, Vol. 16 (2011) No.1, p.47-55.
- [2] Yao M, Zhao M: Unmanned Aerial Vehicle Dynamic Path Planning in an Uncertain Environment, *Robotica*, Vol. 33 (2014) No.3, p.611-621.
- [3] Phung D M, Quach H C, Dinh H T, et al: Enhanced Discrete Particle Swarm Optimization Path Planning for UAV Vision-Based Surface Inspection, *Automation in Construction*, Vol. 81 (2017), p.25-33.
- [4] Na Lin, Jiacheng Tang, Xianwei Li, et al: A Novel Improved Bat Algorithm in UAV Path Planning, *Computers, Materials & Continua*, Vol. 61 (2019) No.1, p.323-344.

- [5] Jain G, Yadav G, Prakash D, et al: MVO-Based Path Planning Scheme with Coordination of UAVs in 3-D Environment, *Journal of Computational Science*, Vol. 37 (2019), p.101016.
- [6] Xiangyin Z, Shuang X, Xiuzhi L: Quantum Behavior-Based Enhanced Fruit Fly Optimization Algorithm with Application to UAV Path Planning, *International Journal of Computational Intelligence Systems*, Vol. 13 (2020) No.1, p.1315.
- [7] Li K, Ge F, Han Y, et al: Path Planning of Multiple UAVs with Online Changing Tasks by an ORPFOA Algorithm, *Engineering Applications of Artificial Intelligence*, Vol. 94 (2020), p.103807.
- [8] Qu C, Gai W, Zhang J, et al: A Novel Hybrid Grey Wolf Optimizer Algorithm for Unmanned Aerial Vehicle (UAV) Path Planning, *Knowledge-Based Systems*, Vol. 194 (2020), p.105530.
- [9] Yaoming Z, Yu S, Anhuan X, et al: A Newly Bio-Inspired Path Planning Algorithm for Autonomous Obstacle Avoidance of UAV, *Chinese Journal of Aeronautics*, Vol. 34 (2021) No.9, p.199-209.
- [10] Ricardo M, D. P G: A Novel Path Planning Optimization Algorithm Based on Particle Swarm Optimization for UAVs for Bird Monitoring and Repelling, *Processes*, Vol. 10 (2021) No.1, p.62-62.
- [11] Yueyi C, Husheng W, Renbin X: Improved Wolf Pack Algorithm for UAV Path Planning Problem, *International Journal of Swarm Intelligence Research (IJSIR)*, Vol. 13 (2022) No.1, p.1-22.
- [12] Yuting W, Yanfei Z, Ailong M, et al: An Accurate UAV 3-D Path Planning Method for Disaster Emergency Response Based on an Improved Multiobjective Swarm Intelligence Algorithm, *IEEE Transactions on Cybernetics*, Vol. 53 (2022) No.4, p.2658-2671.
- [13] Yuwen D: Multi-UAV Search and Rescue with Enhanced A\* Algorithm Path Planning in 3D Environment, *International Journal of Aerospace Engineering*, Vol. 2023 (2023), p.8614117.
- [14] Deng L, Chen H, Zhang X, et al: Three-Dimensional Path Planning of UAV Based on Improved Particle Swarm Optimization, *Mathematics*, Vol. 11 (2023) No.9, p.1987